OCR
Oxford Cambridge and RSA

Friday 19 May 2017 – Morning

AS GCE MATHEMATICS (MEI)

4755/01 Further Concepts for Advanced Mathematics (FP1)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4755/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

5203985880

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

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- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 4 & -3 \\ 2 & 1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} a & 4 \\ 7 & a+3 \end{pmatrix}$ and **I** is the 2×2 identity matrix. Find

(i) $2\mathbf{A} - \mathbf{B} + 3\mathbf{I}$, giving your answer in terms of a, **[3]**

(ii) the value of a for which $\mathbf{AB} = \begin{pmatrix} 3 & -11 \\ 19 & 17 \end{pmatrix}$, **[2]**

(iii) the values of a for which **B** is singular. **[3]**

2 The complex number $2-3\mathrm{j}$ is denoted by z.

(i) Find $|z|$ and $\arg z$. **[2]**

(ii) You are given that $2az + 3z^* = 5 - b\mathrm{j}$, where a and b are real. Find the values of a and b. **[4]**

3 **(i)** Using the standard summation formulae, find an expression for $\sum_{r=1}^{n} (1-2r)^2$ in terms of n. Give your answer in a fully factorised form. **[6]**

(ii) Hence evaluate $\sum_{r=25}^{75} (1-2r)^2$. **[2]**

4 The Argand diagram in Fig. 4 shows a half-line l and a circle C. The circle has centre $3+4\text{j}$.

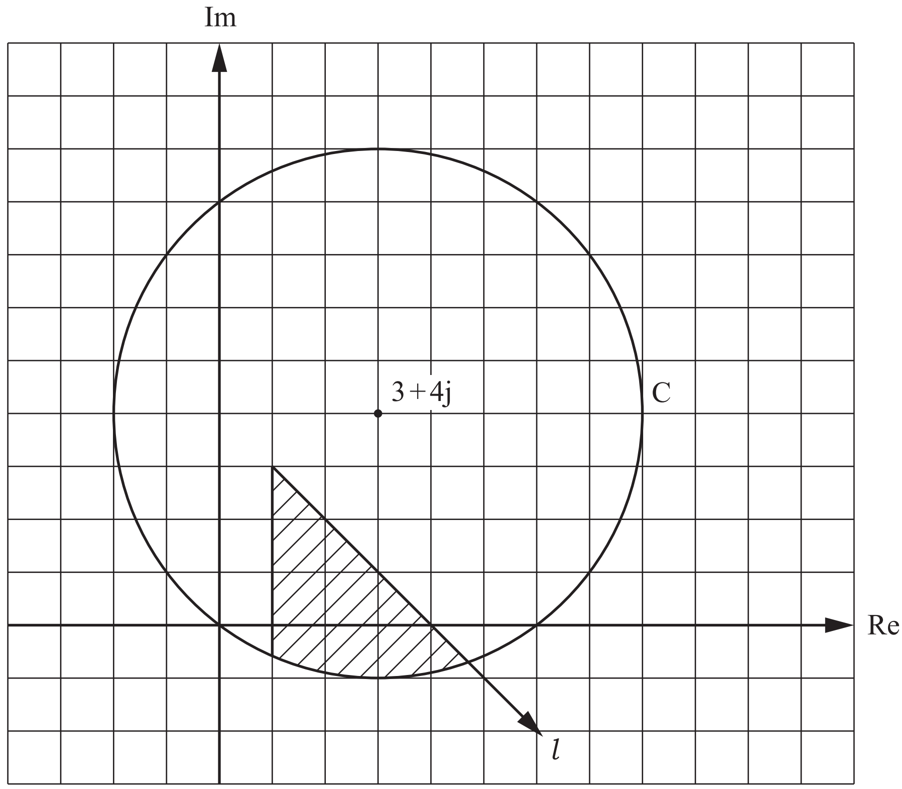

Fig. 4

(i) Write down, in complex number form, the equations of l and C. **[4]**

(ii) Write down inequalities that define the shaded region indicated in Fig. 4, together with its boundaries. **[3]**

5 Prove by induction that $\sum_{r=1}^{n} \frac{r}{2^r} = 2 - \left(\frac{1}{2}\right)^n (2+n)$. **[7]**

Section B (36 marks)

6 A curve has equation $y = \dfrac{ax^2 - 12}{4x^2 + bx - 6}$, where a and b are constants.

(i) Find the coordinates of the point where the curve crosses the y-axis. **[1]**

(ii) You are given that the curve has a vertical asymptote at $x = 2$. Find the value of b and the equation of the other vertical asymptote. **[3]**

(iii) You are given that the curve crosses the x-axis when $x = \pm\sqrt{6}$. Find the value of a and the equation of the horizontal asymptote. **[2]**

(iv) Sketch the curve. **[3]**

(v) Find the set of values for which $y \geqslant 0$. **[3]**

7 **(a)** The roots of the cubic equation $2x^3 - x^2 + 4x + 2 = 0$ are α, β and γ. Find the cubic equation whose roots are 3α, 3β and 3γ, expressing your answer in a form with integer coefficients. **[5]**

(b) A second cubic equation $x^3 + px^2 + qx + r = 0$, where p, q and r are real, has roots that may be written as $a - \lambda$, a and $a + \lambda$.

(i) By considering the sum of the roots show that $2p^3 - 9pq + 27r = 0$. **[4]**

(ii) Given that $p = -6$ and $q = 37$ find the roots of this second cubic equation. **[4]**

8 **(i)** The matrix **P** is given by $\mathbf{P} = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$. Describe fully the geometrical transformation represented by **P**. **[2]**

The matrix **R** is given by $\mathbf{R} = \begin{pmatrix} 2 - 3\sqrt{3} & 3 + 2\sqrt{3} \\ 1 + \sqrt{3} & -1 + \sqrt{3} \end{pmatrix}$.

(ii) Show that the multiplication of **P** and **R** is not commutative. **[2]**

(iii) The transformation represented by **R** is equivalent to the transformation represented by **P** followed by another transformation represented by the matrix **Q**. Find **Q**. **[5]**

(iv) The transformation represented by **Q** is applied to a figure of area 4 square units. Find the area of the transformed figure. **[2]**

END OF QUESTION PAPER

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Duration: 1 hour 30 minutes

4755 01

5204046536

Candidate forename		Candidate surname	

Centre number						Candidate number				

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Section A (36 marks)

1(i)	
1 (ii)	
1 (iii)	

2 (i)

2 (ii)

3 (i)	
3 (ii)	

4 (i)	
4 (ii)	

5

(answer space continued on next page)

5	**(continued)**

Section B (36 marks)

6 (i)	
6 (ii)	
6 (iii)	

6 (iv)	
6 (v)	

7 (a)	

7 (b) (i)	

7(b)(ii)	
	(answer space continued on next page)

7(b)(ii)	(continued)
8 (i)	

8 (ii)

8 (iii)	
8 (iv)	

PLEASE DO NOT WRITE ON THIS PAGE

GCE

Mathematics (MEI)

Unit **4755**: Further Concepts for Advanced Mathematics

Advanced Subsidiary GCE

Mark Scheme for June 2017

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✘	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the

establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

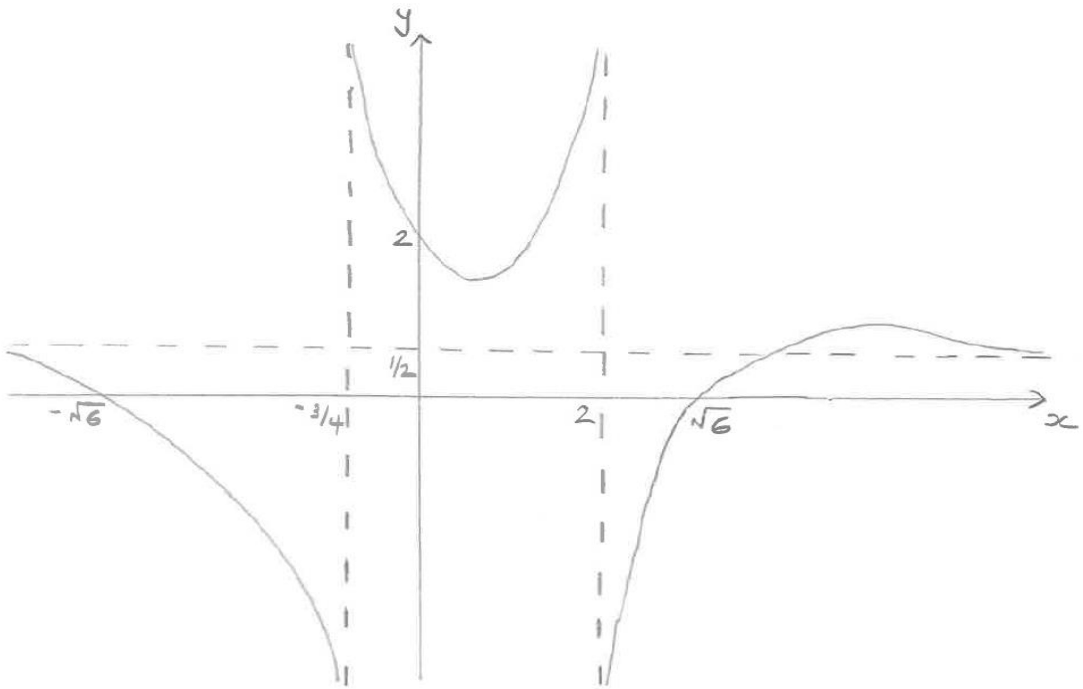
h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question		Answer	Marks	Guidance
1	(i)	$2\mathbf{A}-\mathbf{B}+3\mathbf{I}=\begin{pmatrix}8 & -6\\ 4 & 2\end{pmatrix}-\begin{pmatrix}a & 4\\ 7 & a+3\end{pmatrix}+\begin{pmatrix}3 & 0\\ 0 & 3\end{pmatrix}$	B1	$2\mathbf{A}$ correct
		$=\begin{pmatrix}11-a & -10\\ -3 & 2-a\end{pmatrix}$	B1 B1	B1 any two elements correct, B1 cao
			[3]	
1	(ii)	eg $4a-21=3$	M1	Correct matrix multiplication of $\mathbf{A}$ and $\mathbf{B}$ and equating to corresponding element of $\begin{pmatrix}3 & -11\\ 5 & 17\end{pmatrix}$
		$a=6$	A1	
			[2]	
1	(iii)	$a(a+3)-7(4)=0$	M1*	Attempt at det'nt and equating to 0
		$(a-4)(a+7)=0$	M1dep*	Expand and attempt to solve resulting quadratic
		$a=4$ or $a=-7$	A1	
			[3]	
2	(i)	$\lvert z\rvert=\sqrt{13}$ and $\arg z=-0.983$ (3sf)	B1 B1	Accept $\lvert z\rvert=3.61$ (or better)
			[2]	
2	(ii)	$z^*=2+3\text{j}$	B1	
		$4a+6=5$ and $-6a+9=-b$	M1	Substitute for z and their z^* **and** equate real and imaginary parts
			M1	Attempt to solve equations for both a and b
		$a=-\frac{1}{4}$ and $b=-\frac{21}{2}$	A1	
			[4]	

Question		Answer	Marks	Guidance
3	(i)	$\sum_{r=1}^{n}(1-2r)^2=\sum_{r=1}^{n}1-4\sum_{r=1}^{n}r+4\sum_{r=1}^{n}r^2$	M1*	Separate into three sums (may be implied by later working)
		$=n-2n(n+1)+\frac{2}{3}n(n+1)(2n+1)$	M1*	Use of correct standard results for $\sum r, \sum r^2$
			A1	Correct unsimplified
		$=\frac{1}{3}n\left(3-6(n+1)+2(n+1)(2n+1)\right)$	M1dep*	Attempt to factorise with *n* and a numerical factor
		$=\frac{1}{3}n\left(4n^2-1\right)$	A1	
		$=\frac{1}{3}n(2n-1)(2n+1)$	A1	cao SC if not factorised then M0 B1 for $\frac{4}{3}n^3-\frac{1}{3}n$
			[6]	
3	(ii)	$\sum_{r=25}^{75}(1-2r)^2=\frac{1}{3}(75)(149)(151)-\frac{1}{3}(24)(47)(49)$	M1	Difference of their sum to 75 and their sum to either 24 or 25
		$=544051$	A1 **[2]**	
4	(i)	$C:\lvert z-(3+4\text{j})\rvert=5$	B1 B1	B1 for LHS and B1 for ‘ = 5 ‘
		$l:\arg(z-(1+3\text{j}))=-\frac{\pi}{4}$	B1 B1 **[4]**	B1 for LHS and B1 for ‘ $=-\frac{\pi}{4}$ ‘
4	(ii)	$\lvert z-(3+4\text{j})\rvert\le 5$	B1ft	ft their C
		$-\frac{\pi}{2}\le\arg(z-(1+3\text{j}))\le-\frac{\pi}{4}$	B1ft B1	B1for –π/2, -π/4 and inequalities with whatever they have for *l* in (i) B1 cao
			[3]	

Question		Answer	Marks	Guidance
5		When $n=1$, LHS = $\frac{1}{2}$ and RHS = $2-\left(\frac{1}{2}\right)^1(2+1)=2-\frac{3}{2}=\frac{1}{2}$ so true for $n=1$	B1	
		Assume true for $n=k$ that is $\sum_{r=1}^{k}\frac{r}{2^r}=2-\left(\frac{1}{2}\right)^k(2+k)$	E1	Assume true for $n=k$
		$\sum_{r=1}^{k+1}\frac{r}{2^r}=2-\left(\frac{1}{2}\right)^k(2+k)+\frac{k+1}{2^{k+1}}$	M1*	Add correct term to RHS followed by working
		$=2-\left(\frac{1}{2}\right)^k\left(2+k-\left(\frac{k+1}{2}\right)\right)$	M1dep*	Attempt to factorise with a factor of $\left(\frac{1}{2}\right)^k$ or $\left(\frac{1}{2}\right)^{k+1}$
		$=2-\left(\frac{1}{2}\right)^k\left(\frac{4+2k-k-1}{2}\right)$		
		$=2-\left(\frac{1}{2}\right)^{k+1}\left(2+(k+1)\right)$ or $2-\left(\frac{1}{2}\right)^{k+1}(k+3)$ **with target seen**	A1	cao with correct working
		Therefore **if** it is true for $n=k$ **then** it is true for $n=k+1$.	E1	Dependent on A1 and first E1
		It is true for $n=1$ therefore it is true for $n=1,2,3,\ldots$ (and so is true for all positive integers).	E1	Dependent on B1 and second E1
			[7]	
6	(i)	$(0,2)$	B1	
			[1]	
6	(ii)	$4(2)^2+b(2)-6=0\Rightarrow b=-5$	B1	
		$(4x+3)(x-2)$	M1	Attempt to find the factors of the denominator
		$x=-\frac{3}{4}$	A1	Must be an equation e.g. not 'asymptote $=-\frac{3}{4}$'
			[3]	

Question		Answer	Marks	Guidance
6	(iii)	$a = 2$	B1	
		horizontal asymptote: $y = \frac{1}{2}$	B1	Must be an equation
			[2]	
6	(iv)		B1	3 branches correct – must see a horizontal asymptote with a turning point above it in the right hand branch
			B1	Asymptotes correct and labelled
			B1	Intercepts correct and labelled B0 if more than 3 intercepts
			[3]	
6	(v)	$x \le -\sqrt{6}, \quad -\frac{3}{4} < x < 2, \quad x \ge \sqrt{6}$	B3	One mark for each. Correct inequality signs. Allow 2.45 for $\sqrt{6}$ (B3 then -1 if more than 3 inequalities)
			[3]	

Question		Answer	Marks	Guidance
7	(a)	$w = 3x \Rightarrow x = \frac{w}{3}$	B1	Substitution.
		$\Rightarrow 2\left(\frac{w}{3}\right)^3 - \left(\frac{w}{3}\right)^2 + 4\left(\frac{w}{3}\right) + 2 = 0$	M1	Substitution into cubic
		$\Rightarrow 2w^3 - 3w^2 + 36w + 54 = 0$	A3	correct cubic equation with integer coefficients (minus 1 each error)
			[5]	**SC** For substitution $x = 3w$, ft for a maximum of B0 M1 A1
		OR		
		$\alpha + \beta + \gamma = \frac{1}{2}$ $\alpha\beta + \beta\gamma + \gamma\alpha = 2$ $\alpha\beta\gamma = -1$	M1	Attempt to find sums and products of roots (at least two of three correct)
		Let new roots be k, l, m then $k + l + m = 3(\alpha + \beta + \gamma) = \frac{3}{2}$ $kl + lm + mk = 9(\alpha\beta + \beta\gamma + \gamma\alpha) = 18$ $klm = 27\alpha\beta\gamma = -27$	M1	Attempt to use sums and products of roots of original equation to find all 3 new root relations
		$w^3 - \frac{3}{2}w^2 + 18w + 27 \quad (= 0)$	A2	Correct coefficients (with correct signs) consistent with x^3 coefficient, minus 1 each error
		$\Rightarrow 2\omega^3 - 3\omega^2 + 36\omega + 54 = 0$	A1 [5]	Correct cubic equation with integer coefficients

Question		Answer	Marks	Guidance
7	(b)(i)	$a+\lambda+a+a-\lambda=\pm p$ $a+\lambda+a+a-\lambda=-p$ o.e.	M1 A1	
		$\left(-\frac{p}{3}\right)^3+p\left(-\frac{p}{3}\right)^2+q\left(-\frac{p}{3}\right)+r=0$	M1	Substitute their a into cubic equation and attempt to simplify
		$\Rightarrow 2p^3-9pq+27r=0$	E1	www (answer given)
		OR		
		$a+\lambda+a+a-\lambda=\pm p$ $a+\lambda+a+a-\lambda=-p$ o.e.	M1 A1	
		$q=3a^2-\lambda^2$ $-r=a^3-a\lambda^2$		
		$(2p^3-9pq+27r=)\ 2(-3a)^3-9(-3a)(3a^2-\lambda^2)+27(a\lambda^2-a^3)$	M1	p,q and r in terms of a and λ and substituted
		$=0$	E1	www
			[4]	
7	(b)(ii)	$a=2$	B1	
		$p=-6, q=37 \Rightarrow r=-58 \Rightarrow 2(2-\lambda)(2+\lambda)=58$ or $q=37 \Rightarrow 37=3(2^2)-\lambda^2$	M1	Valid method to find λ^2 or λ
		$\lambda=(\pm)5\text{j}$	A1	
		Other roots are $2-5\text{j}$ and $2+5\text{j}$	A1	
		OR		
		$a=2$	B1	
		$(x-2)(x^2-4x+29)$ by inspection or long division	M1,A1	2 terms in quadratic correct for M1
		Other roots are $2-5\text{j}$ and $2+5\text{j}$	A1	
			[4]	

Question		Answer	Marks	Guidance
8	(i)	**P** represents a rotation about the origin through an angle of 60 degrees (or π/3 radians) in a clockwise direction	**B1** **B1** **[2]**	Rotation about origin 60° clockwise, oe e.g. - 60°
8	(ii)	e.g. $\frac{1}{2}\left(2-3\sqrt{3}\right)+\frac{\sqrt{3}}{2}\left(1+\sqrt{3}\right)$ and $\frac{1}{2}\left(2-3\sqrt{3}\right)-\frac{\sqrt{3}}{2}\left(2\sqrt{3}+3\right)$ $\frac{1}{2}\left(5-2\sqrt{3}\right)\neq-2-3\sqrt{3}$ therefore PR ≠ RP	**M1** **E1** **[2]**	Calculate a corresponding pair of elements for **PR** and **RP** with at least one correct Matrix products unequal explicitly stated but must have attempted a corresponding pair from each matrix
8	(iii)	**QP = R** o.e. e.g. **Q** = **RP**$^{-1}$ $\mathbf{P}^{-1}=\begin{pmatrix}1/2 & -\sqrt{3}/2\\ \sqrt{3}/2 & 1/2\end{pmatrix}$ **Q** = **RP**$^{-1}$ evaluated $\mathbf{Q}=\begin{pmatrix}4 & 6\\ 2 & -2\end{pmatrix}$ OR $\begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}1/2 & \sqrt{3}/2\\ -\sqrt{3}/2 & 1/2\end{pmatrix}=\begin{pmatrix}2-3\sqrt{3} & 3+2\sqrt{3}\\ 1+\sqrt{3} & -1+\sqrt{3}\end{pmatrix}$ $a/2-b\sqrt{3}/2=2-3\sqrt{3}$, $c/2-d\sqrt{3}/2=1+\sqrt{3}$ o.e. $\mathbf{Q}=\begin{pmatrix}4 & 6\\ 2 & -2\end{pmatrix}$	**B1** **B1** **M1** **A1 A1** **B1** **B1** **M1** **A1 A1** **[5]**	Correct order of operations Inverse of **P** Attempt to find $\mathbf{RP}^{-1}$ or $\mathbf{P}^{-1}\mathbf{R}$ A1 any two correct entries, A1 cao Correct order of operations a correct equation in *a* and *b* and a correct equation in *c* and *d* solving for *a*,*b*,*c* and *d* A1 any two correct entries, A1 cao
8	(iv)	Area of transformed figure = 4 × |(4(-2) – 2(6))| = 80	**M1** **A1** **[2]**	Multiplying determinant of **Q** by 4 cao from a correct Q **SC** 4(4+10√3) from $P^{-1}R$; M1A0

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4755 Further Concepts for Advanced Mathematics (FP1)

General Comments:

Many good scripts were presented but by question 8 it was clear that some candidates were pressed for time. Many candidates struggled to demonstrate mathematical precision with basic algebraic manipulation, even in the standard simplifications involved in questions 1, 2 and 3. A somewhat reckless disregard for brackets and for correct notation often made many answers technically incorrect and sometimes led to errors by the candidate in following work. Candidates need to understand what it means to show method; to show the mathematical reasoning behind the calculation, rather than just the result from the calculator, and to show logical steps one after another. Incorrect manipulation of negative numbers was particularly noticeable. One of the requirements at this level is a correct understanding of mathematical terminology, notation and language coupled with and an ability to write with precision.

Comments on Individual Questions:

Question 1
1(i) A quite surprising number of errors were seen in evaluating 2**A**. Sometimes this was not shown and could not be inferred from a wrong final result. A few candidates calculated **A** x **A**. The biggest problem was incorrect arithmetic. A considerable proportion of candidates made errors with the addition and subtraction of negative numbers and consequently failed to calculate correctly all 4 elements of the matrix.

1(ii) Nearly all candidates earned full marks here.

1(iii) This was also done well though some demonstrated an inability to solve a simple quadratic equation, merely substituting the value a = 4.
A significant number of candidates did not know what a singular matrix was.
Some produced the matrix $\begin{pmatrix} 4 & 4 \\ 7 & 7 \end{pmatrix}$ with no indication of how it came about.

Question 2
2(i) The value of $|z|$ was nearly always correct. Most candidates gave fully correct answers, but arg(z) was quite often written as positive. Some expressed it in degrees or as a reflex angle, neither earning the mark.

2(ii) Finding the value of *a* was completed in most cases but there were a lot of arithmetical errors in finding the value of *b*, demonstrating an inability to manipulate negative numbers. A surprising number of candidates went from the correct
– *b* = 10.5 to then write *b* = 10.5. Many failed to evaluate $-6\times-\frac{1}{4}$ correctly, or wrote $2a\times-3\mathrm{j}$ as $-6\mathrm{j}$, or wrote $3\times3\mathrm{j}$ as $6\mathrm{j}$. Candidates who carefully wrote out the equations for the real parts and the imaginary parts were least likely to make mistakes.

Question3
3(i) This question was mostly well done, with many candidates showing good algebraic skills, although there were some very clumsy final expressions. The aim should be to clear fractions from factors, obtaining a single numerical factor. The initial split was achieved apart from a few who failed to expand the square correctly. A fairly frequent mistake was to leave $\sum 1$ as 1, a costly error as factorisation was impossible. Another more common error was to leave the result as $\frac{1}{3}n(4n^2-1)$ where the difference of two squares was not noticed. Candidates should write n

firmly as a multiplier of a fraction or in the numerator because, for example, $\frac{1}{3}n$ was often written to look like $\frac{1}{3n}$.

3(ii) "Hence" in the question required that the summation be carried out using the previous result. Work which did not show this explicitly earned no marks. Several made the mistake of subtracting the sum of 25 terms from the sum of 75 terms.

Question 4
4(i) The solutions to this question were very mixed with many candidates unable to write the equations of the circle and the line in complex number form. The circle was more often given correctly but we often saw $|C-3-4\mathrm{j}|$ or (z – 3 – 4j) or $|3+4\mathrm{j}|=5$.. In dealing with l, 'arg' was often omitted or replaced by modulus, and $\arg(l-1-3\mathrm{j})=-\frac{\pi}{4}$ was also seen. Some candidates anticipated part (ii) and introduced an inequality. Some gave a reflex angle, not always accurately.

4(ii) The question clearly stated that the boundaries were included but many candidates used strict inequalities. Upper and lower bounds of the argument written $-\frac{\pi}{4}\le\ldots\le-\frac{\pi}{2}$ were not infrequently seen.

Question 5
A substantial minority demonstrated an inability to cope with indices. This meant that, for many, this question was not about demonstrating the concept of proof by induction, but about index manipulation.
The algebra proved too difficult for most. In particular, a minus sign outside the bracket was easily the most common error here.
Bad handwriting meant that indices often got confused, e.g. the index numbers were written in line with everything else and then became coefficients instead of indices. In the denominator for example, 2^{k+1} often became 2^k+1 or $2(k+1)$ or even $2k+1$ in later work.
Some multiplied out and created very complicated expressions, often carrying through terms such as 1^{-2k} for several lines seemingly unaware that it could be simplified. Rough working was interspersed between lines of their deductive steps without making it clear what they were doing. We often saw a whole line multiplied by 2^{k+1} and later if we were lucky, divided by 2^{k+1} with no explanation given. The most successful were those who started by creating two fractions with clear denominators of 2^k and 2^{k+1}, and developed the work from there.
Many candidates write the last statements down even if they have not done previous work. They need to be aware that this wastes their time because of the way this question is marked, where previous steps need to be correct.
Apart from these points, it was good to see that a great many candidates had learned the appropriate introductions to the proof, and where they were successful in the algebraic argument, followed up with the crucial inductive argument, earning full marks. There were still a few candidates who insisted that their result showed that "n = k+1 is true".

Question 6
6(i) There were few problems here, but some write simply $y=2$ instead of $(0,2)$ or $x=0, y=2$.

6(ii) In this part the most frequent error was in not writing the asymptote as an equation so "asymptote = –3/4" was often seen.

6(iii) Similarly, here, "asymptote = 1/2" lost a mark.

6(iv) A common error in drawing the graph was to have the right hand part of the curve approach the y = ½ asymptote from below.
Many candidates lost a mark because they did not clearly demonstrate that they knew how the right hand part of the curve behaved. They tried to cross the asymptote and at the same time move parallel to it so that there was no indication of a turning point followed by a point of inflection. Too many candidates believe that a "sketch" can be as roughly drawn as they like. It would be good to see a ruler used for the axes. The curve and the scales cannot be exact, which is why all the intercepts must be labelled, likewise the asymptotes. Some candidates omitted some or all of these, in particular the labelling at ½ on the y-axis, where the line itself was not labelled. That said, many excellent sketches were seen with all salient points and asymptotes clearly shown.

6(v) Most candidates answered this part well. There were the usual errors with inequality signs the wrong way round, using '<' instead of ' $\leq$ ' or using 'y' as the variable instead of x.
Trying to save time or space by writing $-\sqrt{6} \geq x \geq \sqrt{6}$ resulted in a careless loss of 2 marks.

Question 7
7(a) This part was answered very well by those who used the substitution 'x = w/3' with most going on to gain all 5 marks. Marks were usually lost either by forgetting to multiply the numerical term by 27 or by forgetting to equate the expression to zero.
The candidates who used the sums and products of roots fared less well, one of the reasons being that there was more opportunity to go wrong. Sign errors were common and so "sum of roots = -1/2" was often seen as was "product of roots = 1". Also some who correctly had the values 1/2 , 2 and –1 then went on to multiply all of these by 3 rather than by 3, 9 and 27 respectively. The signs had again to be carefully considered when transferring the new coefficients into the new equation.

7 (b) (i) Nearly all candidates were able to gain 2 marks by writing '$3a = -p$' or an equivalent. Very rarely the sum of the roots was equated to p not to $-p$.
A variety of methods were used. Those that went on with the simpler substitution 'x = –p/3' usually gained the next 2 marks with little trouble.
Many did not realise that this was the simplest method and instead derived expressions for q and r in terms of a and λ. . They then substituted these values into the given expression and showed that all of the terms cancelled out. There was more scope for error using this method and it was more time consuming. Again the sign errors cropped up in the expression for r with many writing $r = a^3 - a\lambda^2$ instead of the correct version $-r = a^3 - a\lambda^2$. After substitution, another source of error was over hasty simplification of the expression, with several multiplications of minus signs required. Very often the errors cancelled out to obtain a "correct" result, but the explanation mark was given for a solution "without wrong working".
It is worth pointing out that while the first method of substituting a root into an equation gives an expression equal to zero straight away, in the second method, using the given expression as the starting point, this equality to zero should not be claimed before the analysis demonstrates that zero results.

7 (b) (ii) In a few cases it appeared that candidates believed that the cubic equation in part (a) was relevant in part (b). Throughout part (b) there was a commonly seen change of notation with α appearing consistently instead of a. Indeed a was here sometimes confused with the coefficient of x^3, giving $a = 1$. Most candidates used $p = -6$ to obtain a . They then obtained a quadratic equation for λ using either the value of q and the sum of the root products in pairs, or using the product of the roots after calculating r. When solving $\lambda^2 = -25$, most candidates gave 5j as their solution rather than $\pm 5\text{j}$. Fortunately both were then used in giving the complex roots. Many candidates could not believe that the other two roots would involve complex numbers and ignored their negative λ^2. Some of the solutions offered found r from the earlier result and produced a cubic equation. It was expected that candidates would demonstrate their ability to solve this, rather than resorting to a calculator.

Question 8

8(i) Plenty of correct answers were seen. Most recognised that the matrix represented a rotation. There was confusion over the amount. Many candidates described the rotation as 60^o anticlockwise, while several thought that the angle was 30^o. A few chose 45^o or even 90^o. Many unfortunately failed to specify the centre of the rotation.

8(ii) Many candidates seemed unaware that in order to prove that matrices are not equal it is sufficient to demonstrate that two corresponding elements are not equal. They wasted valuable time completely evaluating both products **PR** and **RP**, which was by no means straightforward. Incorrect matrix products were common, but the marking for this was generous.
A common error was to calculate 2 corresponding elements and then state that because these two elements were different then the matrix multiplication was not commutative. 'Unequal elements, so not commutative' was insufficient. 'Unequal elements, so that **PR** is not the same as **RP'** on the other hand shows the meaning of "multiplication of **P** and **R** is not commutative". The candidates who had found the full matrix product in each case usually achieved this explanation mark, a trade-off for the time spent!

8(iii) Most candidates knew that **R** was equal to QP. About half the candidates then aimed to use $\mathbf{P}^{-1}$ and the other half set up equations involving the unknown elements of **Q**. The former group very often went on to calculate $\mathbf{P}^{-1}\mathbf{R}$ instead of $\mathbf{RP}^{-1}$. A method mark was available for the multiplication either way, but not for some candidates who thought that $\mathbf{R}^{-1}$ was involved.
The equations set up by the second group were usually solved successfully. Many candidates did not realise that the rational and irrational parts could in this instance be easily separated.

8(iv) Many candidates knew that the determinant supplied the scale factor for the area. Candidates who clearly stated a value for their determinant and used this as the multiplier for the area 4 earned at least one mark. If their matrix **Q** was correct, and their new area correct, they earned both the marks. If the determinant was not clearly stated, a correct **Q** and correct new area earned both the marks, but anything else did not.

Oxford Cambridge and RSA

Unit level raw mark and UMS grade boundaries June 2017 series

For more information about results and grade calculations, see www.ocr.org.uk/ocr-for/learners-and-parents/getting-your-results

AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award

GCE Mathematics (MEI)

				Max Mark	a	b	c	d	e	u
4751	01	C1 – MEI Introduction to advanced mathematics (AS)	Raw	72	63	58	53	49	45	0
			UMS	100	80	70	60	50	40	0
4752	01	C2 – MEI Concepts for advanced mathematics (AS)	Raw	72	55	49	44	39	34	0
			UMS	100	80	70	60	50	40	0
4753	01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	54	49	45	41	36	0
4753	02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753	82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4754	01	C4 – MEI Applications of advanced mathematics (A2)	Raw	90	67	61	55	49	43	0
			UMS	100	80	70	60	50	40	0
4755	01	FP1 – MEI Further concepts for advanced mathematics (AS)	Raw	72	57	52	47	42	38	0
			UMS	100	80	70	60	50	40	0
4756	01	FP2 – MEI Further methods for advanced mathematics (A2)	Raw	72	65	58	52	46	40	0
			UMS	100	80	70	60	50	40	0
4757	01	FP3 – MEI Further applications of advanced mathematics (A2)	Raw	72	64	56	48	41	34	0
			UMS	100	80	70	60	50	40	0
4758	01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	56	50	44	37	0
4758	02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758	82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4761	01	M1 – MEI Mechanics 1 (AS)	Raw	72	57	49	41	34	27	0
			UMS	100	80	70	60	50	40	0
4762	01	M2 – MEI Mechanics 2 (A2)	Raw	72	56	48	41	34	27	0
			UMS	100	80	70	60	50	40	0
4763	01	M3 – MEI Mechanics 3 (A2)	Raw	72	58	50	43	36	29	0
			UMS	100	80	70	60	50	40	0
4764	01	M4 – MEI Mechanics 4 (A2)	Raw	72	53	45	38	31	24	0
			UMS	100	80	70	60	50	40	0
4766	01	S1 – MEI Statistics 1 (AS)	Raw	72	61	55	49	43	37	0
			UMS	100	80	70	60	50	40	0
4767	01	S2 – MEI Statistics 2 (A2)	Raw	72	56	50	45	40	35	0
			UMS	100	80	70	60	50	40	0
4768	01	S3 – MEI Statistics 3 (A2)	Raw	72	63	57	51	46	41	0
			UMS	100	80	70	60	50	40	0
4769	01	S4 – MEI Statistics 4 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4771	01	D1 – MEI Decision mathematics 1 (AS)	Raw	72	52	46	41	36	31	0
			UMS	100	80	70	60	50	40	0
4772	01	D2 – MEI Decision mathematics 2 (A2)	Raw	72	53	48	43	39	35	0
			UMS	100	80	70	60	50	40	0
4773	01	DC – MEI Decision mathematics computation (A2)	Raw	72	46	40	34	29	24	0
			UMS	100	80	70	60	50	40	0
4776	01	(NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	58	53	48	43	37	0
4776	02	(NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776	82	(NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
4777	01	NC – MEI Numerical computation (A2)	Raw	72	55	48	41	34	27	0

			Max Mark	a	b	c	d	e	u
		UMS	100	80	70	60	50	40	0
4798	01 FPT - Further pure mathematics with technology (A2)	Raw	72	57	49	41	33	26	0
		UMS	100	80	70	60	50	40	0

GCE Statistics (MEI)

			Max Mark	a	b	c	d	e	u
G241	01 Statistics 1 MEI (Z1)	Raw	72	61	55	49	43	37	0
		UMS	100	80	70	60	50	40	0
G242	01 Statistics 2 MEI (Z2)	Raw	72	55	48	41	34	27	0
		UMS	100	80	70	60	50	40	0
G243	01 Statistics 3 MEI (Z3)	Raw	72	56	48	41	34	27	0
		UMS	100	80	70	60	50	40	0

GCE Quantitative Methods (MEI)

			Max Mark	a	b	c	d	e	u
G244	01 Introduction to Quantitative Methods MEI	Raw	72	58	50	43	36	28	0
G244	02 Introduction to Quantitative Methods MEI	Raw	18	14	12	10	8	7	0
		UMS	100	80	70	60	50	40	0
G245	01 Statistics 1 MEI	Raw	72	61	55	49	43	37	0
		UMS	100	80	70	60	50	40	0
G246	01 Decision 1 MEI	Raw	72	52	46	41	36	31	0
		UMS	100	80	70	60	50	40	0

Level 3 Certificate and FSMQ raw mark grade boundaries June 2017 series

For more information about results and grade calculations, see www.ocr.org.uk/ocr-for/learners-and-parents/getting-your-results

Level 3 Certificate Mathematics for Engineering

				Max Mark	a*	a	b	c	d	e	u
H860	01	Mathematics for Engineering		This unit has no entries in June 2017							
H860	02	Mathematics for Engineering									

Level 3 Certificate Mathematical Techniques and Applications for Engineers

				Max Mark	a*	a	b	c	d	e	u
H865	01	Component 1	Raw	60	48	42	36	30	24	18	0

Level 3 Certificate Mathematics - Quantitative Reasoning (MEI) (GQ Reform)

				Max Mark	a	b	c	d	e	u
H866	01	Introduction to quantitative reasoning	Raw	72	54	47	40	34	28	0
H866	02	Critical maths	Raw	60*	48	42	36	30	24	0
		*Component 02 is weighted to give marks out of 72	Overall	144	112	97	83	70	57	0

Level 3 Certificate Mathematics - Quantitive Problem Solving (MEI) (GQ Reform)

				Max Mark	a	b	c	d	e	u
H867	01	Introduction to quantitative reasoning	Raw	72	54	47	40	34	28	0
H867	02	Statistical problem solving	Raw	60*	41	36	31	27	23	0
		*Component 02 is weighted to give marks out of 72	Overall	144	103	90	77	66	56	0

Advanced Free Standing Mathematics Qualification (FSMQ)

				Max Mark	a	b	c	d	e	u
6993	01	Additional Mathematics	Raw	100	72	63	55	47	39	0

Intermediate Free Standing Mathematics Qualification (FSMQ)

				Max Mark	a	b	c	d	e	u
6989	01	Foundations of Advanced Mathematics (MEI)	Raw	40	35	30	25	20	16	0